Problem solving lecture

- Caveat: We will neglect friction for now but will consider it in lecture 6. For problems involving motion in free fall, we will use $g = 9.8\,m\,s^{-2}$. Enjoy!

~~1~~ Dynamics and Kinematics:

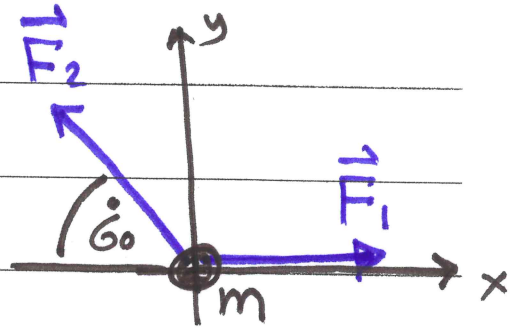
A force (F) accelerates a mass m from (vi) to (vf) in (t)s. Knowing F, vi, vf and t, what is m?

F is constant, thus a is constant "or uniform".

$F = ma \Rightarrow m = \frac{F}{a}$, $v_f = v_i + at \Rightarrow a = (v_f - v_i)/t$

$\therefore m = \frac{Ft}{(v_f - v_i)}$. Do problem 4.7 and examples: 4.2 and 4.3

2 The mass m (2 kg) is subject to two forces $\vec{F}_1$ ($|\vec{F}_1| = 20N$) and $\vec{F}_2$.



If the acceleration of the mass is along the y-axis, then $|\vec{F}_2|$ is:

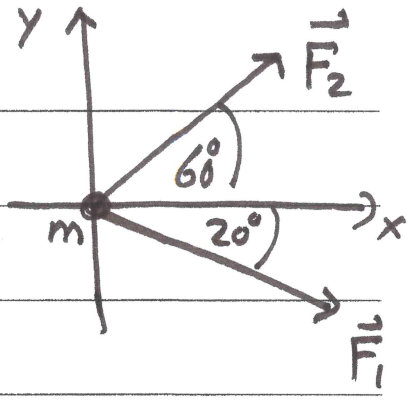
Right off: $\sum \vec{F}_x = 0 \Rightarrow |\vec{F}_1| = |\vec{F}_2| \cos(60^\circ)$.

- There is no need to determine $\vec{a}$!
- Recall page 4 in lecture 4, item 5.

3 Past exam:

Given: $|\vec{F}_1| = 5N$, $|\vec{F}_2| = 8N$

$m = 0.3\,kg \Rightarrow$



Determine the acceleration of m:-

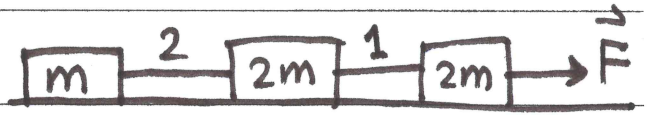
$|\vec{a}| = 34\,ms^{-2}$, $\theta = 31^\circ$ w.r.t. the +ve x-axis's.

- Do ex 4.9, problem 4.28 (b): $|\vec{F}| = 14N$, $\theta = 51^\circ$.

4 PHY 105 and Tension !

The force exerted by a string, rope, cable, wire, spring rod, thread, cord, or nerve is called the tension in the ... (you name it!). It's a contact force.

- Past exam: Take $m = 2kg$ and $|\vec{F}| = 25N$, what is the tension in wire 2?



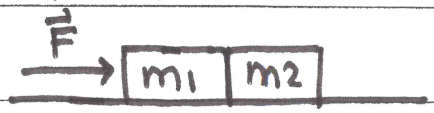
The 3 masses have the same acceleration because they are connected. The "driving" force for the whole system is F, $F = 5ma$. Wire 2 exerts tension T_2 on the mass m $\Rightarrow T_2 = ma = m\left[\frac{F}{5m}\right] = 5N$

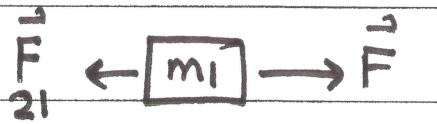
Qwiz: What is T_1? Is it 3ma?

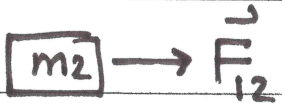
Do ex 4.11, 4.12.

5 Past exam: One block pushes another!

Two blocks of masses m_1 and m_2, with $m_1 > m_2$ are placed in contact with each other. A constant force $\vec{F}$ is applied to m_1 as shown.







Determine the force exerted by m_1 on m_2, i.e., $\vec{F}_{12}$.

$\Rightarrow$ The two blocks must experience the same acceleration because they are in contact with each other and remain in contact throughout the motion.

$\therefore F = (m_1+m_2)a \rightarrow a = F/(m_1+m_2)$.

F is the driving force of the system (external).

F_{12} and F_{21} (the contact forces) are internal forces.

Apply Newton's 2nd law to m_2: $F_{12} = m_2 a = \left[\frac{m_2}{m_1+m_2}\right] F$

- Notice that $F_{12} < F$ ✓.
- Validate: $F_{21} \stackrel{?}{=} F_{12} \Rightarrow$ 2nd law to m_1:

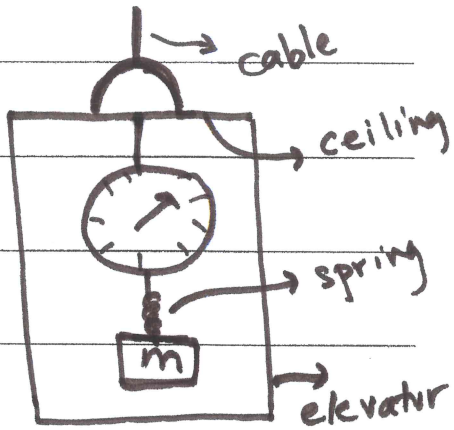
$$F - F_{21} = m_1 a \Rightarrow F_{21} = F - m_1 a = F - m_1\left(\frac{F}{m_1+m_2}\right) = F_{12}$$ ✓

- Do problems 4.11, 4.33
  Do examples 4.6, 4.7

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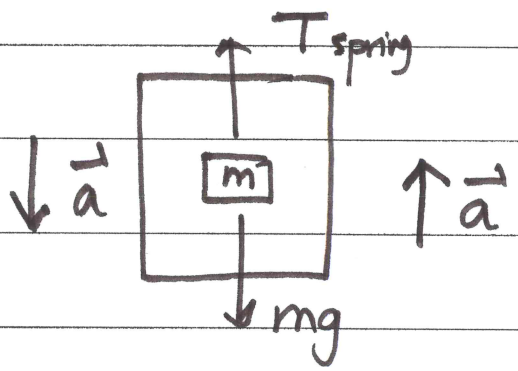
6 Weighing a "fish" in an elevator: apparent weight!

You; a PHY 105 student @JU, weigh a fish of mass m on a spring scale attached to the ceiling of an elevator as shown.



- Determine the reading on the scale if the elevator accelerates either upward or downward.

⇒ let $\vec{a}$ = acceleration of the elevator.



$$\Rightarrow +T + (-mg) = m\vec{a}$$

$$\Rightarrow T = mg + m\vec{a}$$

$$\Rightarrow T = mg\left[1 + \frac{\vec{a}}{g}\right] \quad \text{——— (*)}$$

(i) ∴ When the elevator accelerates upward (ascends), then $\vec{a} = +a$, where a is determined by the engine of the elevator ⇒ Thus:

$$T = mg\left[1 + \frac{a}{g}\right] \Rightarrow T > mg,$$ thus the spring scale reads a value greater than the fish's weight; the apparent weight!

(ii) When the elevator accelerates downward (descends), then $\vec{a} = -a$, then $T = mg\left[1 - \frac{a}{g}\right] \Rightarrow T < mg$, thus the spring scale reads a value less than the weight of the fish; the apparent weight!

(iii) If the elevator is either at rest or moving at constant velocity, then eq (*) reads: $\vec{a} = 0$, $\Rightarrow T = mg$; the weight of the fish.

(iv) Suppose that the elevator cable breaks, what happens to the reading of the scale?

The acceleration of the elevator is determined by the engine, and if the cable breaks, then the elevator becomes a freely falling object: $\vec{a} = \vec{g}$.

Eq (*) reads $T = mg\left[1 + \frac{-g}{g}\right] = zero$, the scale reading is zero; i.e., the fish "appears" to be weightless!

- The "moral" of this "story" is that a gangster will first sell you a fish while the elevator is accelerating upward, and then buy it from you while the elevator is accelerating downward!!

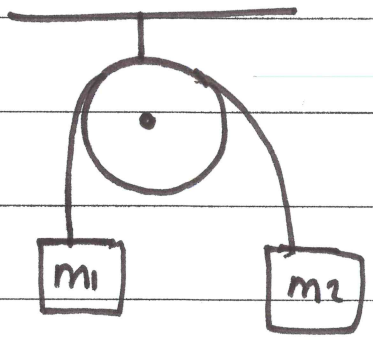
• Evaluate the scale readings for a 40 N fish if the elevator moves with an acceleration $\vec{a} = \pm 2 m s^{-2}$.

⇒ If $\vec{a}$ is upward ⇒ T = 48.2 N.

⇒ If $\vec{a}$ is downward ⇒ T = 31.8 N.

• Do problem 4.18 ⇒ $a = 0.44 m s^{-2}$, do ex 4.8.

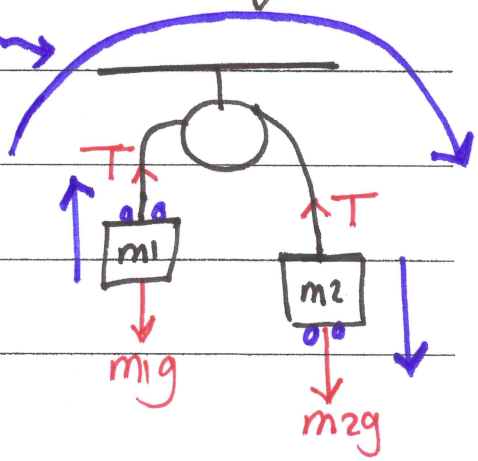
7 The Atwood machine:



The arrangement of two masses (m_1, m_2) hung vertically over a frictionless pulley of negligible mass is called Atwood machine, named after George Atwood (1745–1807).

Let $m_2 > m_1$, determine the acceleration of the two masses and the tension in the string connecting both.

Notice the direction of the flow ~~~>



∴ Sign convention: with the flow +ve

opposite the flow −ve

∴ For m_2: $+m_2 g + (-)T = (+) m_2 a$

For m_1: $+T + (-m_1 g) = (+) m_1 a$

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↑ +ve
↓ -ve

- If you want to use our sign convention then you must be very careful. Notice that if m_2 accelerates downward, m_1 accelerates upward, therefore, for consistency with signs, one reads:

For m_2: $+T+(-)m_2g=(-)m_2a$

For m_1: $+T+(-)m_1g=(+)m_1a$ → identical to eqs. page 6.

⇒ Solve for the acceleration:-

$$a=\left(\frac{m_2-m_1}{m_2+m_1}\right)g$$ → 7th grade algebra!

⇒ Solve for the tension $T=\left(\frac{2m_1m_2}{m_1+m_2}\right)g$.

- Do dimensional analysis for the values of a and T.
- The sign of a depends on the relative masses:
  if $m_2>m_1$, a is +ve, corresponding to our flow
  If $m_2<m_1$, a is -ve, indicating the opposite flow!
- If $m_1=m_2$, $a=0$; the system is balanced; $T=mg$.
- If $m_2 \gg m_1 \Rightarrow a\simeq g$, m_2 simply falls freely.
- Past exam: If you double both masses, the resulting a will be:

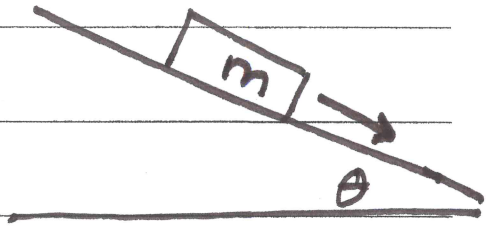
[twice as large, half as large, the same, one-quarter as large, four times as large.]

- Do ex 4:13.

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8 Sliding down a hill .... Hmm!

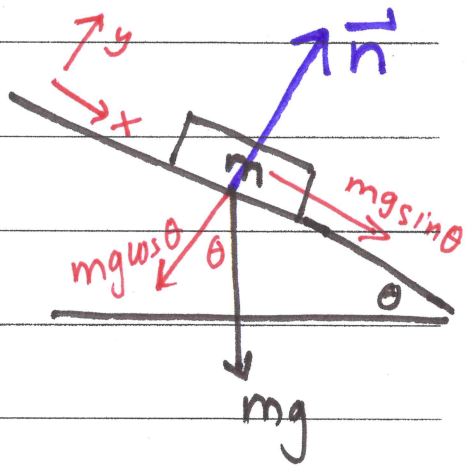
A car of mass m is on a driveway inclined at an angle θ as shown.



① Determine the acceleration of the car.

Recall the definition of the normal force; $\vec{n}$. Normal in this's context means perpendicular to the incline.

∴ The driving force is $mg\sin\theta$.



$\Rightarrow mg\sin\theta = ma$ —— (1) x-axis along the incline.

$\Rightarrow n = mg\cos\theta$ —— (2) y-axis perpendicular to the incline

∵ the car is in equilibrium in the y-axis, i.e., it is in contact with the incline during the motion.

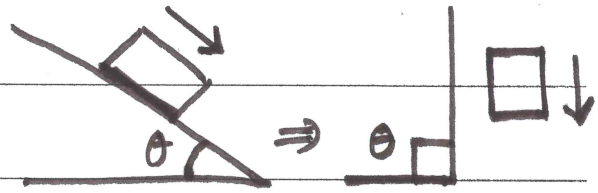
$\Rightarrow$ Eq (1) reads: $\cancel{m}g\sin\theta = \cancel{m}a \Rightarrow a = g\sin\theta$ —(3)

(i-1) Eq (3): a is independent of the mass of the car. Thus, the acceleration of the car equals that of the car key!

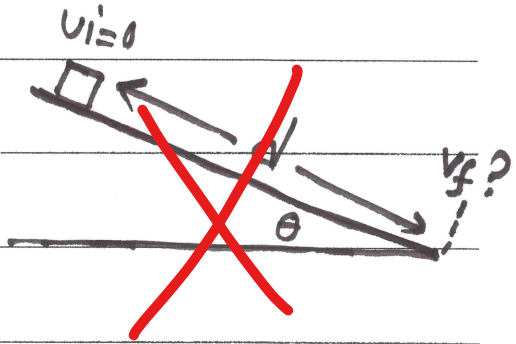
(i-2) Eq(2): $n = mg\cos\theta \Rightarrow$ the normal force is not equal in magnitude to the weight of the object, as discussed in page 7- lecture 4.

(i-3) What previously solved problem does this situation become if $\theta = 90°$?

The inclined plane becomes vertical, thus eq(3) becomes $a = g$ and the car is an object in free fall! Notice also that eq (2) gives us $n = 0$. This is consistent with the car falling downward next to the vertical plane, in which case there is NO contact force between the car and the plane.



(ii) If the car is released from rest at the top of the hill and is sliding down a distance d to reach the bottom of the hill, as shown.



- How long does it take to slide the distance d, and what is it's speed as it arrives there?

Recall problem [1]: This part of problem [8] belongs to kinematics (chapter 2) rather than to dynamics (chapter 4).

Eq (3) shows that a is constant, $a = g\sin\theta$ ✓.

- $\Delta\vec{x} = \vec{v}_i t + \frac{\vec{a}t^2}{2} \Rightarrow d = 0 + \frac{at^2}{2} \rightarrow t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2d}{g\sin\theta}}$
- $v_f^2 = v_i^2 + 2\vec{a}\Delta\vec{x} \Rightarrow v_f = \sqrt{2gd\sin\theta}$
- $\vec{v}_f = \vec{v}_i + \vec{a}t \Rightarrow v_f = \dots$, which eq is safer to use?
- notice that v_f and t are mass independent ✓.

- Do problems 4.52 and 4.53.
- Take this advice: note that the mass, ~~m~~, dropped out of our answer for the acceleration (eq 3). With such cancellation, there is a big advantage of working with the algebraic equations and putting in the numbers only at the end.

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