CHAPTER 7 - Hypothesis testing

Wednesday, 26 November 2025 9:00 AM

★ Our main goal is to make a decision about a hypothesis using data from a sample regarding a population

One Sample Problem

⇒ when you use one sample to make a claim about one population

Two Sample Problem

⇒ You compare 2 different samples (of 2 different populations) to decide if their populations differ — not discussed in this Chapter

★ ★ ★

H_0: the null hypothesis which is to be tested

H_1: the alternative hypothesis that competes with H_0

e.g.

$H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$ or $\mu < \mu_0$ or $\mu \neq \mu_0$ → one - sided / two sided → $\mu \neq \mu_0$

★ ★ ★

Decisions :)

★ Accepting H_0 and H_0 is true ✓ good decision

★ Accepting H_0 and H_0 is false, H_1 is true ✗ bad decision (type II error)

★ Rejecting H_0 and H_0 is true ✗ bad decision (type I error)

★ Rejecting H_0 and H_0 is false, H_1 is true ✓ good decision

⇒ type 1 error : P(reject H_0 | H_0 is true)
↳ the significance level of a test
the Probability of type I error is α

⇒ type 2 error : P(accept H_0 | H_0 is false)
the Probability of type 2 error is β

⇒ power test = $1 - \beta$ = $1 - P(\text{accept } H_0 \mid H_0 \text{ is false})$
Probability of making the correct decision = $P(\text{reject } H_0 \mid H_0 \text{ is false})$

⇒ Type 2 error happens when the true population mean isn't equal to μ_0 (H_0 is false) but the test fails to detect the difference (we accept H_0 therefore it is false)

↳ So, β is the Probability of missing a true difference between the true mean μ and μ_0

high β
⇒ the difference between μ and μ_0 is small so it is more likely to miss the difference and make a wrong decision
⇒ the power test is small

low β
⇒ the difference between μ and μ_0 is big and easy to detect so it is less likely to make an error
⇒ the power test is big

★ The aim of hypothesis testing is to use statistical tests to make α and β as small as possible (meaning that the test makes errors as rarely as possible)

tools that use sample data to check if a hypothesis about the population is likely to be true

in statistical tests we can use

- **the critical value**
  ⇒ you compare the test statistic with the critical value to decide whether it is accepted or rejected
  (uses only data)
- **P-value**
  ⇒ you find the Probability of getting a result rejected in comparison with our test statistic
  using them is called normal theory method

TO FIND THE TEST STATISTIC

★ keep in mind that for a Population with known standard deviation, we use **Z-test** and for a population with unknown standard deviation ($n \geq 200$) we use **t-test**

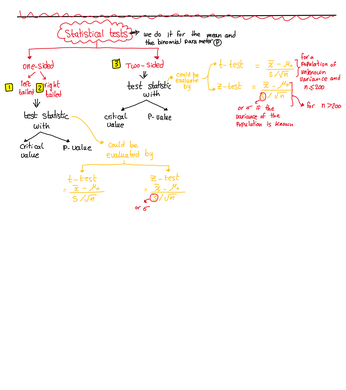
① state the hypotheses H_0 and H_1

② Choose a significance level α (if not mentioned in the question we use 0.05 (the standard))

③ Z - test or t - test

$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ $t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$

↳ $\bar{X}$ is the mean of the Sample



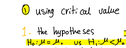
A list of all ★ ★ ★ rules 8)

A Statistical testing of the mean μ

$H_0: \mu = \mu_0$

1 For left-tailed

① using critical value



1. the hypotheses $H_0: \mu = \mu_0$ vs $H_1: \mu < \mu_0$
2. the test statistic

for a population with unknown variance + $n \leq 200$	for a population with known variance or a population of unknown variance + $n > 200$
t-test : $\frac{\bar{X} - \mu_0}{S/\sqrt{n}}$	Z-test = $\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$ or $\frac{O}{\sigma/\sqrt{n}}$

3. find the critical value

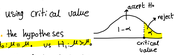
using t-test	using Z-test
critical value = $-t_{n-1,\,1-\alpha}$	critical value= $-Z_{1-\alpha}$
we find it from the table	we find it from the normal table

4. Compare the test statistic with the critical value

test statistic > critical value	test statistic < critical value
reject H_0	Accept H_0

2 For right-tailed

① using critical value



1. the hypotheses $H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$
2. the test statistic
   ★ the Same as before
3. find the critical value

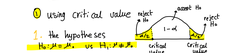
using t-test	using Z-test
critical value = $t_{n-1,\,1-\alpha}$	critical value= $Z_{1-\alpha}$
we find it from the table	we find it from the normal table

4. Compare the test statistic with the critical value

test statistic > critical value	test statistic < critical value
reject H_0	Accept H_0

3 For two sided

① using critical value



1. the hypotheses $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$
2. the test statistic
   ★ the same as before
3. find the critical value

using t-test	using Z-test
critical values $t_{n-1,\,\alpha/2}$ and $t_{n-1,\,1-\alpha/2}$	critical values $Z_{\alpha/2}$ and $Z_{1-\alpha/2}$
Find them from the t-table	find them from the Z value

4. Compare the test statistic with the critical value

-critical value < test statistic < critical value	test statistic > critical value or test statistic < -critical value
accept H_0	reject H_0

② using P- value

1. the hypotheses $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$
2. the test statistic
   ★ the Same as before
3. Find P- value

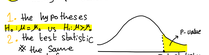
test statistic > 0	test statistic < 0
① $\frac{1}{2}$ P-value / Test statistic	② $\frac{1}{2}$ P-value / Test statistic
P-value = $2P(t > \text{test statistic})$	P-value = $2P(t < \text{test statistic})$
P-value = $2P(Z > \text{test statistic})$	P-value = $2P(Z < \text{test statistic})$

4. Compare the P-value with the significance level α

P-value > α	P-value < α
⇒ accept H_0	⇒ reject H_0

② using P- value

1. the hypotheses $H_0: \mu = \mu_0$ vs $H_1: \mu > \mu_0$ / $H_1: \mu < \mu_0$ / $H_1: \mu \neq \mu_0$
2. the test statistic
   ★ the Same as before
3. Find P- value



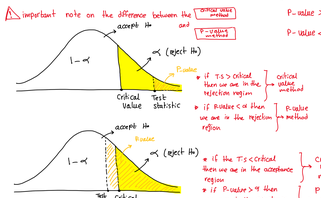
using t	using Z
P-value = $P(t > \text{test statistic})$	P-value = $P(Z > \text{test statistic})$
from the t table	from the Z table
P-value = $P(t < \text{test statistic})$	P-value = $P(Z < \text{test statistic})$
from the t table	from the Z table

4. Compare the P-value with the significance level α

P-value > α	P-value < α
⇒ accept H_0	⇒ reject H_0

★ note that P-value > α is always accept H_0
P-value < α is always reject H_0
} for all cases

⚠ important note on the difference between the critical value method and the P-value method



★ if T.S > critical then we are in the rejection region → critical value method
★ if P-value < α then we are in the rejection region → P-value method

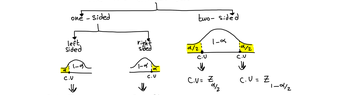
★ if the T.S < critical then we are in the acceptance region → critical value method
★ if P-value > α then we are in the acceptance region → P-value method

→ this concept applies to all tests that use the Critical value method and the P-value method

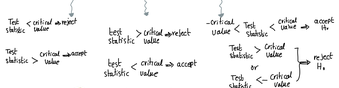
B Statistical testing for the binomial Parameter P (Proportion)

① using the critical value

1. the hypotheses $H_0: P = P_0$ vs $H_1: P > P_0$ → right-sided / $H_1: P < P_0$ → left-sided / $H_1: P \neq P_0$ → two - sided
2. test statistic = $\frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}}$ ★ only Z - test is used
3. Find the critical value



4. Decision making



② using the P- value

1. the hypotheses $H_0: P = P_0$ vs $H_1: P > P_0$ → right-sided / $H_1: P < P_0$ → left-sided / $H_1: P \neq P_0$ → two - sided
2. find the test statistic
   ★ the Same as before
3. Find the P-value



= $P(Z < \text{test statistic})$ = $P(Z > \text{test statistic})$ = $2P(Z < \text{test statistic})$ = $2P(Z > \text{test statistic})$

4. Decision making

P-value > α ⇒ accept
P-value < α ⇒ reject

★ ★ ★

- if P-value < α ⇒ statistically significant } when using P-value
  ↓
  H_0 rejected
- if P-value > α ⇒ not statistically significant
  ↓
  H_0 accepted

★ if α isn't mentioned in the question we use 0.05

- if H_0 is rejected ⇒ statistically significant } when using the critical value
- if H_0 is accepted ⇒ not statistically significant

? when is a one - sided test more appropriate than a two sided test ?

- a one - sided test is better than a two - sided test because it has more Power (it is easier to reject H_0 based on a finite Sample (if H_1 is actually true)

★ A decision whether to use one - sided or two - sided test must be made before the data analysis

★ ★ ★

Sample size estimation based on CI width

→ in the confidence interval $\bar{X} \pm Z_{1-\alpha/2} * \frac{S}{\sqrt{n}}$ ← S: if the variance is known we use σ

we have :-

the length of CI = UB - LB

$L = 2 Z_{1-\alpha/2} * \frac{S}{\sqrt{n}}$

could be using the t - destribution for $n \leq 200$ and unknown variance

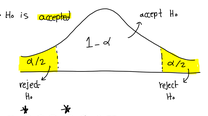
$n = \left(2 Z_{1-\alpha/2} * \frac{S}{L}\right)^2$ ⇓ $n = \left(2 t_{n-1,\,1-\alpha/2} * \frac{S}{L}\right)^2$

⚠ After finding n we must round up
2.01 → 3
2.9 → 3

The relation ship between hypothesis testing and confidence interval

two - sided test case

★ If $(1-\alpha)$ 100% CI contains μ_0 ⇒ H_0 is accepted
★ If $(1-\alpha)$ 100% CI doesn't contain μ_0 ⇒ H_0 is rejected



the significance of a P-value

